Design And Implementatio Of Multiloop Controllers For MIMO System

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Abstract: The objective of this paper is to design controllers for the TITO (Two Inputs and Two Outputs) system, the nonlinear system considered is quadruple tank process. In order to design the controller initially the work is started from the mathematical modeling. Using the basic law of conservation of mass the differential equations are obtained. The nonlinear differential equations are linearized using jacobian matrix and the state space model is determined. The state space model is converted into process transfer function matrix. Using the process transfer function matrix, the three controllers responses for both minimum and non-minimum phase is obtained. The designed controller values are determined using SIMULINK model in Matlab. The designed controller values are multi loop PID controller, Decentralized PID controller and IMC PID controller. The simulation response of tank1 and tank2 are achieved using MATLAB /Simulink. Here in the experimental setup the controller is selected from the performance error criteria. The controller with the less error and good step response tracking is considered as the best controller.

Keywords: Quadruple tank process, Minimum phase, Non-minimum phase Different Controller, Error Criteria.

I. Introduction

In process control industries, the basic need is to The basic aim of process control is to retain the process operations at required condition carefully and resourcefully, while fulfilling the quality of product and ecology. The quadruple tank process is a MIMO system which is multi input and multi output. The quadruple tank process experimental description and modeling are explained in the below session. In this paper the design of multi loop, decentralized and IMC controller are depicted. The quadruple tank process experimental set up is explained in the later session. The minimum and non-minimum phase of the quadruple tank process response of tank1 and tank2 are shown.

2.1 Transfer function matrix

II. Modeling And Design Of Controllers

The schematic diagram of quadruple tank process is shown in Figure 1. In order to design the controller, mathematical modeling of quadruple tank process should be done. The mathematical modeling represents the whole system in differential equation. From the basic law of conservation of mass and Bernoulli's law the mathematical model is completed. The differential equations of quadruple tank process is represented in equation (1) to (4). The quadruple tank process is a nonlinear system since the equation (1) to (4) contain the square root function. The nonlinear differential equations are linearized using Jacobian matrix. The general representation of Jacobian matrix for A=(4*4) matrix and B=(4*2) matrix is shown in equation (5) and (6). By using the values tabulated in TableI and by substituting the tabulated values in linearized Jacobian matrix results in state space model. The transfer function matrix represented in equation (7) & (8) is obtained from state space model.



Fig.1 Schematic Diagram of Quadruple Tank Process

The open loop response of tank1 and 2 of quadruple tank process for minimum and non-minimum phase is obtained using transfer function matrix equation (7) & (8). The block diagram is shown in Figure2 and the simulation of open loop response for tank1 and 2 are shown in Figure3 and Figure4. The simulation of open loop response for tank1 and 2 shows uncontrolled response, which should be tuned to desired set point by using controller. In this paper, the multi loop and decentralized controller designed.

$$\frac{dh_1}{dt} = \left(\frac{\gamma 1 k 1 v 1}{A1}\right) + \left(\frac{a 3 \sqrt{2 g h 3}}{A1}\right) - \left(\frac{a 1 \sqrt{2 g h 1}}{A1}\right)$$
(1)

$$\frac{dh_2}{dt} = \left(\frac{\gamma 2 k 2 v 2}{A2}\right) + \left(\frac{a 4 \sqrt{2 g h 4}}{A2}\right) - \left(\frac{a 2 \sqrt{2 g h 2}}{A2}\right)$$
(2)

$$\frac{dh_3}{dt} = \left(\frac{(1 - \gamma 2) k 2 v 2}{A3}\right) - \left(\frac{a 3 \sqrt{2 g h 3}}{A3}\right)$$
(3)

$$\frac{dh4}{dt} = \left(\frac{(1-\gamma 1)k1v1}{A4}\right) - \left(\frac{a4\sqrt{2gh4}}{A4}\right)$$
(4)

Where

A1,A2,A3,A4-C.S Area of the Tank1,2,3,4

h1,h2,h3,h4-Height of Water in Tank1,2,3,4

v1,v2-Velocity of Flow through Pump1and2

k1,k2,kc-Pump Constant

γ1,γ2-Valve Ratio

a1,a2,a3,a4- C.S Area of Outlet Pipe of Tank1,2,3,4

g-Acceleration due to Gravity

$$\mathbf{A} = \frac{\delta f}{\delta h}(\eta, \upsilon) = \begin{pmatrix} \delta f1/\delta h1 & \delta f1/\delta h2 & \delta f1/\delta h3 & \delta f1/\delta h4 \\ \delta f2/\delta h1 & \delta f2/\delta h2 & \delta f2/\delta h3 & \delta f2/\delta h4 \\ \delta f3/\delta h1 & \delta f3/\delta h2 & \delta f3/\delta h3 & \delta f3/\delta h4 \\ \delta f4/\delta h1 & \delta f4/\delta h2 & \delta f4/\delta h3 & \delta f4/\delta h4 \end{pmatrix} (5)$$
$$\mathbf{B} = \frac{\delta f}{\delta u}(\mathbf{h}, \mathbf{u}) = \begin{pmatrix} \delta f1/\delta u1 & \delta f1/\delta u2 \\ \delta f2/\delta u1 & \delta f2/\delta u2 \\ \delta f3/\delta u1 & \delta f2/\delta u2 \\ \delta f4/\delta u1 & \delta f2/\delta u2 \end{pmatrix}$$
(6)

Table.I PARAMETERS SPECIFICATION

Parameters	Minimum Phase	Non Minimum
	Value	Phase
A1=A2=A3=A4	63.585cm/2	63.585cm/2
a1=a2	1.246cm/2	1.246cm/2
Operating point of h1	12.4 &12.7 cm	12.6 & 13.0
&h2		
Operating point of h3	1.8 & 1.4 cm	4.8 & 4.9
&h4		
k1=k2	3 3 ^{cm³} 3 3 ^{cm³}	3.14^{cm^3} 3.29^{cm^3}
	vs vs vs	vs vs vs
γ1 & γ2	0.7 &0.6	0.43, 0.34

$$\mathbf{A} = \begin{pmatrix} -0.1232 & 0 & 0.2038 & 0 \\ 0 & -0.3668 & 0 & 0.2310 \\ 0 & 0 & -0.2038 & 0 \\ 0 & 0 & 0 & -0.2310 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 0.0367 & 0 \\ 0 & 0.0316 \\ 0 & 0.0211 \\ 0.0157 & 0 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The above state space matrix is Minimum phase. The state space matrix for non-minimum phase is given below.

$$\mathbf{A} = \begin{pmatrix} -0.1223 & 0 & 0.1243 & 0 \\ 0 & -0.1204 & 0 & 0.1231 \\ 0 & 0 & -0.1243 & 0 \\ 0 & 0 & 0 & -0.1231 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 0.0212 & 0 \\ 0 & 0.0176 \\ 0 & 0.0341 \\ 0.0281 & 0 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{pmatrix}$$
$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The transfer function matrix is determined from state space model. The process transfer function matrix for minimum phase and non-minimum phase is denoted by G_{P-} and G_{P+}

$$G_{p}(s) = \begin{pmatrix}
 G_{11}(s) & G_{12}(s) \\
 G_{21}(s) & G_{22}(s)
 \end{pmatrix} = \begin{pmatrix}
 \frac{0.0184}{(S+0.1232)} & 0.0022 \\
 \frac{0.0018}{(S+0.1232)(S+0.2038)} \\
 \frac{0.0158}{(S+0.3668)}
 \end{pmatrix}$$

$$G_{p+}(s) = \begin{pmatrix}
 G_{11}(s) & G_{12}(s) \\
 G_{21}(s) & G_{22}(s)
 \end{pmatrix} = \begin{pmatrix}
 \frac{0.0106}{(S+0.1223)} & 0.0021 \\
 \frac{0.0106}{(S+0.1223)} & 0.0021 \\
 \frac{0.0018}{(S+0.1223)(S+0.1243)} \\
 \frac{0.0018}{(S+0.1204)(S+0.1231)} & 0.0088 \\
 \frac{0.0088}{(S+0.1204)}
 \end{pmatrix}$$

$$(8)$$

From the obtained process transfer function matrix, the interactive open loop process response for minimum and non-minimum phase is shown in below Figure 2 & 3.







III. Design Of Multi Loop PID Controller

The multi loop PID controller values for both minimum and non-minimum phase is found using Cohen and coon method.

3.1 Multi Loop PID Controller for Minimum Phase

The quadruple process is a MIMO system which has minimum phase. The minimum phase in quadruple tank process system is defined as when the fraction of liquid entering the lower tank is less when compared to upper tank. The minimum phase is a stable condition, where the design of multi loop controller is simple when compared to non minimum phase. The multi loop PID controller value for minimum phase value is tabulated below.

Table.II Multi Loop PID Controller For Minimum Phase

controllers	Proportional Gain	Integral Gain	Derivative Gain
1	16.7080	3.57	1.89
2	62.9699	2.02	1.69

The response of tank1 & tank2 for the above tabulated PID controller value are show in Figure 4&5.



Fig.5 Tank2 Response of Multi Loop PID Controller2 for Minimum Phase

3.2 Multi Loop PID Controller for Non-Minimum Phase

The non-minimum phase of quadruple tank process is defined as when the fraction of liquid entering the lower tank is greater than the upper tank. The design of controller for non-minimum phase is complex as it is unstable and also due to the presence of transmission zero. The multi loop PID controller value for non-minimum phase is tabulated in TableIII. The response of tank1 & tank2 are shown in below figures.



Table.III MULTI LOOP PID CONTROLLER FOR NON- MINIMUM PHASE





Fig.7 Tank2 Response of Multi Loop PID Controller2 for Non-minimum phase

IV. Design Of Decentralized PID Controller

The decentralized controller commonly encountered in many industrial processes in order to have effective control over the process. the decentralized controller eliminates the process loop interactions by using ideal decoupler. The design of decentralized PID Controller for minimum and non-minimum phase is dealt in further session.

4.1. Design of Decoupler matrix

The decoupler eliminates the interaction within process. The equation (7) & (8) represents the process transfer function matrix for minimum and non-minimum phase. The interaction compensator matrix (decoupler) is obtained by using equation (9). The element within the interaction compensator matrix is obtained using equations (10) and (11).

$$g_{i1}(S) = -\frac{1}{G_{11}(S)}$$

$$g_{i2}(S) = -\frac{G_{21}(S)}{G_{22}(S)}$$
(10)
(10)
(11)

The $g_{i1}(S)$ and $g_{i2}(S)$ are decoupler matrix for minimum phase is expressed in equation (12) and (13).

$$g_{i1}(S) = -\frac{(0.1196)}{(S+0.2038)}$$
(12)

$$g_{i2}(S) = -\frac{(0.1139)}{(S+0.2310)}$$
(13)

The decoupler matrix for non-minimum phase is expressed in equation (14) and (15).

$$g_{i1}(S) = -\frac{(0.1981)}{(S+0.1243)}$$
(14)

$$g_{i2}(S) = -\frac{(0.1932)}{(S+0.1231)}$$
(15)

The two independent decoupled SISO (Single Input Single Output) system with gains $G_1(S)$ and $G_2(S)$ are obtained from equation (14) and (15).

$$G_{1}(S) = \begin{bmatrix} G_{11}(S) - \frac{G_{12}(S)G_{21}(S)}{G_{22}(S)} \end{bmatrix}$$
(16)
$$G_{2}(S) = \begin{bmatrix} G_{22}(S) - \frac{G_{12}(S)G_{21}(S)}{G_{11}(S)} \end{bmatrix}$$
(17)

The expression for $G_1(S)$ and $G_2(S)$ determined using relations (14) and (15) are given in equation (16) and (17). Using the equation (16) and (17), the decentralized PID controller values for minimum phase are determined.

$$G_{1}(S) = \begin{bmatrix} \frac{0.0184S^{2} + 8 \times 10^{-3}S - 1.6946 \times 10^{-3}}{S^{3} + 0.558S^{2} + 0.1007S + 5.8027 \times 10^{-3}} \end{bmatrix} (18)$$

$$G_{2}(S) = \begin{bmatrix} \frac{0.0158S^{2} + 6.86988 \times 10^{-3}S - 5.2896 \times 10^{-4}}{S^{3} + 0.8016S^{2} + 0.2066S + 0.0173} \end{bmatrix} (19)$$

The equation (20) and (21) gives the decentralized PID controller value for non-minimum phase.

$$\begin{split} G_1(S) &= \left[\frac{0.0106S^2 + 2.6224 * 10^{-3}S - 2.435 * 10^{-4}}{S^3 + 0.3697S^2 + 0.0456S + 0.0018} \right] \ (20) \\ G_2(S) &= \left[\frac{0.0088S^2 + 2.1771 * 10^{-3}S - 2.0215 * 10^{-4}}{S^3 + 0.3678S^2 + 0.045S + 0.0018} \right] \ (21) \end{split}$$

4.2 Decentralized PID Controller for Minimum Phase

The quadruple process is a MIMO system which has minimum phase. The minimum phase in quadruple tank process system is defined as when the fraction of liquid entering the lower tank is less when compared to upper tank. The minimum phase is a stable condition, where the design of multi loop controller is simple when compared to non minimum phase. The decentralized PID controller value for minimum phase value is tabulated below.

 Table.IV.DECENTRALIZED PID CONTROLLER FOR MINIMUM PHASE

controllers	Proportional Gain	Integral Gain	Derivative Gain
1	19.9392	3.9501	32.9825
2	55.9944	33.8833	19.5002

The response of tank1 & tank2 for the above tabulated PID controller value are show in Figure 8&9.



4.3 Decentralized PID Controller for Non-Minimum Phase

The non-minimum phase of quadruple tank process is defined as when the fraction of liquid entering the lower tank is greater than the upper tank. The design of controller for non-minimum phase is complex as it is unstable and also due to the presence of transmission zero. The decentralized PID controller value for non-minimum phase is tabulated in TableV. The response of tank1 & tank2 are shown in below figures.



 Table.V.DECENTRALIZED
 PID CONTROLLER FOR NON- MINIMUM PHASE





Fig.11 Tank2 Response of Decentralized PID Controller2 for Non-Minimum Phase

V. Design Of Internal Model Controller

The internal model controller commonly encountered in many industrial processes in order to have effective control over the process. The tuning of IMC is easy when compared to standard feedback controller design. The design of IMC PID Controller for minimum and non-minimum phase is dealt in further session.

5.1 IMC Design Procedure

The IMC design procedure has been generalized to the following steps.

 $\widetilde{G}_{p}(S)$

Develop a process model $\tilde{g}_p(S)$. Factor the process model into invertible and noninvertible portions.

$$= \tilde{g}_{p+}(S)\tilde{g}_{p-}(S)$$

This factorization is performed so that the resulting controller will be stable. Form the idealized IMC controller. The ideal internal model controller is the invertible portion of the process model.

$$\widetilde{Q}(S) = \widetilde{g}_{p-1}^{-1}(S)$$
(23)

Add a filter to make the controller proper.

(22)

$$Q(S) = \tilde{g}_{n-1}^{-1}(S)f(S)$$

(24)

(25)

(26)

if it is most desirable to track step set point changes, the filter transfer function usually has the form

$$f(S) = \frac{1}{(2S+1)}$$

The closed loop IMC controller is determined by

$$g_{c}(S) = \frac{q(S)}{(1 - \check{g}_{P}(s)q(S))}$$

The standard feedback structure equivalent to IMC is shown in below fig.



Fig.12 IMC Structure

5.2 IMC PID Controller for Minimum Phase

The quadruple process is a MIMO system which has minimum phase. The minimum phase in quadruple tank process system is defined as when the fraction of liquid entering the lower tank is less when compared to upper tank. The minimum phase is a stable condition, where the design of multi loop controller is simple when compared to non minimum phase. The IMC PID controller value for minimum phase value is tabulated below.

Table.VI. IMC PID CONTROLLER FOR MINIMUM PHASE

Controllers	Proportional Gain	Integral Gain	Derivative Gain
1	116.7080	23.7918	26.9293
2	92.1532	44.4998	28.3099

The response of tank1 & tank2 for the above tabulated PID controller value are show in Figure13&14.



Fig.13 Tank1 Response of IMC PID Controller1



5.3 IMC PID Controller for Non-Minimum Phase

The non-minimum phase of quadruple tank process is defined as when the fraction of liquid entering the lower tank is greater than the upper tank. The design of controller for non-minimum phase is complex as it is unstable and also due to the presence of transmission zero. The IMC PID controller value for non-minimum phase is tabulated in TableVII. The response of tank1 & tank2 are shown in below figures.



 Table.VII.
 IMC
 PID CONTROLLER FOR NON- MINIMUM PHASE







VI. Results And Discussion

The experimental set up is shown below. The experimental set up contains the four vertical tanks with respective drain valves for outlet. The experimental set up contains four panels namely instrumentation power supply which provides 230V AC, 50Hz input supply to two pumps, there is two signal conditioning cum thyristor actuator panels which provides the cosine firing angle to the pumps in order to operated the two control valves. There is a V to I converter (Voltage to Current) panel and one computer interface panel through which the experimental set up is connected with PC. The PC acts as an user interface between human and the experimental set up. In PC the designed controller values are implemented and the response from the experimental set up is obtained.



Fig.17 QTP Experimental Setup

The closed loop step response characteristics of multi loop, decentralized PID, IMC PID controller of tank1 & tank2 for both minimum and non-minimum phase is tabulated below.

Controllers	Closed Loop Characteristics	Minimum Phase		Non-minimum Phase	
		Tank1	Tank2	Tank1	Tank2
	Rise Time(Sec)	5.06	2.3	29.1	1.62
Multi Loop	Overshoot(%)	4.22	1.37	1.92	4.92
PID	Settling Time(Sec)	15.3	3.55	145	23.3
Decentralized	Rise Time(Sec)	6.15	2.15	28.9	28
PID	Overshoot(%)	0.284	2.86	3.71	4.27
	Settling Time(Sec)	11.9	6.03	147	143
IMC PID	Rise Time(Sec)	1.16	1.67	8.83	2.27
	Overshoot(%)	1.27	1.02	8.01	12.4
	Settling Time(Sec)	2.01	2.71	103	208

Table. VIII. CLUSED LOOP STEP RESPONSE CHARACTERISTICS OF VARIOUS PID CONTROLLE	Table.VIII.	CLOSED	LOOP STEP	RESPONSE	CHARACTERISTICS	OF VARIOUS PIL	O CONTROLLER
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The closed loop step response characteristics of three controllers for minimum and non-minimum phase are represented using graph. The closed loop step response characteristics like rise time, overshoot and settling time are shown individually. The rise time of all three controllers for minimum and non-minimum phase is shown in Fig. 18 & Fig. 19. using graph.

Rise Time Vs Types of Controllers for Minimum Phase



Fig.18. Rise Time of Three Controllers for Minimum Phase



Rise Time Vs Types of Controller for Non-minimum Phase

Time (Seconds) Fig.19. Rise Time of Three Controllers for Non-Minimum Phase



Fig.20. Overshoot of Three Controllers for Minimum Phase



Overshoot Vs Types of Controller

Fig.21. Overshoot of Three Controllers for Non-Minimum Phase



Fig.22. Settling Time of Three Controllers for Minimum Phase



Fig.23. Settling Time of Three Controllers for Non-Minimum Phase

VII. Conclusions

The various controllers are designed and the best controller is determined by performance error criteria. From the performance error criteria table, the multi loop PID controller has the high error when compared with other two controllers. The reason for high error in multi loop PID controller is due to interactions among the loop. The decentralized PID controller gives better response and it is an excellent step response tracking. The essential part of decentralized controller is decoupler which acts as an interaction compensator. From the closed loop step response characteristics tabulation we conclude that the non-minimum phase of quadruple process system takes longer time than minimum phase. The IMC PID controller is the best controller. The advantage of using IMC is, it is an advanced controller which is more reliable and gives better stability in the process. The performance error criteria for both phases are tabulated below. The performance error criteria considered are IAE (Integral Absolute Error), ITAE (Integral Time Absolute Error), ISE (Integral Square Error)

Controllers	Error criteria	Error Minimum		Non-minimum Phase	
	criteria	Tank1	Tank2	Tank1	Tank2
	IAE	5.02	1.12	44.90	1.32
Multi Loop PID	ITAE	56.83	6.50	1777.25	34.66
	ISE	1.51	0.37	31.84	0.337
Decentralized	IAE	2.43	1.11	41.04	40.95
PID	ITAE	10.8	5.43	900	895
	ISE	1.01	0.42	36.96	36.81
IMC PID	IAE	1.13	0.74	35.91	2.63
	ITAE	10.74	3.81	909.72	34.00
	ISE	0.19	0.24	32.61	0.64

	TABLE IX.	PERFORMANCE ERROR CRITERIA
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The various designed PID controllers value are implemented in the experimental setup (four tank system/ quadruple tank process) and the response is obtained. The responses of different controllers are show in below figures for both minimum and non-minimum phase. The performance error criteria values for the designed controllers are obtained from the SIMULINK model simulation responses which are show in graph.

The graphical representation of performance error criteria of the three controllers for minimum and nonminimum phase are shown individually for tank1 and tank2 which is shown in Fig 24. to 27. In order to determine the best controller, from the simulation result the values are implemented in the experimental set up and the real time response should reach the specified desired response which is shown in below Figures.



Fig.24. Comparison of Controllers with Performance Error Criteria of Tank1 for Minimum Phase







Tank1-Nonminimum Phase



Fig.26. Comparison of Controllers with Performance Error Criteria of Tank1 for Non-Minimum Phase Performance Error Criteria Vs Type of Controllers for Tank2-Nonminimum Phase



Types of Controller

Fig.27. Comparison of Controllers with Performance Error Criteria of Tank2 for Non-Minimum Phase



Fig.28. (Multi loop PID Controller) Experimental Setup Simulation Response of Tank2- Minimum Phase



Fig.29. (Multi loop PID Controller) Experimental Setup Simulation Response of Tank2 Non-Minimum



Fig.30. (Decentralized PID Controller) Experimental Setup Simulation Response of Tank2 -Minimum Phase



Fig.31. (Decentralized PID Controller) Experimental Setup Simulation Response of Tank2 Non-Minimum Phase



Fig.32. (IMC PID Controller) Experimental Setup Simulation Response of Tank2 -Minimum Phase



Fig.33. (IMC PID Controller) Experimental Setup Simulation Response of Tank2 Non-Minimum Phase

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